Curriculum S. 12
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ONTARIO
Department of Education

COURSES OF STUDY

GRADES XI and XII

MATHEMATICS

These revised courses, introduced in September 1952, replace those contained in Circular H.S. 12, printed in 1940, 1942, and 1948.

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COURSES OF STUDY

For

Grades XI and XII

In

Collegiate Institutes, High, Vocational, and Continuation Schools

MATHEMATICS

General Observations

1. The courses in Algebra and Geometry outlined below are the result of the experimental work which has been carried on in certain classes of Grades XI and XII during the school years 1950-51 and 1951-52 in order to explore the possibility of distributing more equitably over Grades XI, XII and XIII the weight of prescribed work in the courses leading to the Grade XIII Departmental Examinations in Mathematics.

2. It is the intention that the modifications which have been made in the existing courses shall lighten to some extent the programme in Grade XIII Algebra and shall result in a more adequate preparation for the study of Trigonometry and Statics. It is also hoped that greater attention to graphs will facilitate later work in Analytic Geometry.

3. Pupils who have completed successfully the general course Mathematics of Grades IX and X should be able to cope with the Grades XI and XII courses as revised. Further, with the development of the programme of options in the various grades, the number of pupils attempting these courses without an adequate foundation should progressively decrease.

4. It will be noted that certain items in the courses are designated as "optional." It is important that these be included in the work taken with above-average classes or in the assignments given to the better students within a class-group of varying ability. In other words, it is recommended not only that the assignment should be on an extended rather than a daily basis, but that the technique of graded, or minimum and maximum, assignments should be followed and that the more difficult examples or those which stress merely manipulative dexterity should usually be assigned only to the abler students and reviewed with them on an individual or a group, rather than a class, basis. It goes without saying that all students must, however, reach a suitable minimum standard in order to be recommended for their graduation diplomas.

5. An appreciable amount of class time may be saved, particularly on the material of sections 1 to 3 of the Algebra and also throughout the course in Geometry, by the procedure suggested in the foregoing paragraphs and the deletions mentioned in the revised courses of study.

6. The order in which the courses in Algebra and Geometry are taken in Grades XI and XII is left, as in the past, to the discretion of the local

authorities. Something is to be said for each arrangement. It might be mentioned, however, that when Geometry is taken in Grade XI and Algebra in Grade XII, the transition from the Mathematics of Grade X to the Geometry of Grade XI and from the Algebra of Grade XII to the Mathematics of Grade XIII is somewhat smoother. Accordingly, a number of schools are adopting this plan.

7. The attention of teachers is drawn to the Departmental memorandum "Suggestions regarding the revised Course in Geometry." This may be of assistance to teachers in planning their work for the year.

Outline of the Courses of Study

ALGEBRA

Section 1. **Algebraic Notation**

Review of the four fundamental operations.

Extension of multiplication and division to include binomial and trinomial multipliers and divisors. Problems involving fractional and literal coefficients.

Inexact division. Verification and checking.

Practice in the use of general numbers.

Section 2. **Equations of the First Degree**

Review of equations of the first degree in one unknown and extension to include equations with literal coefficients.

Equations of the first degree in two unknowns, including those with literal coefficients.

Graphs of linear functions and graphical solutions of pairs of linear equations in two unknowns.

It is important that the idea of function be introduced early and gradually developed throughout this and the Grade XIII course.

Equations of the first degree in three unknowns.

Problems solved by means of equations of the first degree.

Section 3. **Factoring and Its Applications**

Review of all types of factoring, with oral and written practice on straight-forward examples arranged in miscellaneous collections. The use of class time on artificial or unduly complicated examples should be avoided.

Use of the Factor Theorem in factoring such expressions as $x^3+5x^2-2x-24$.

Application of factoring to finding H.C.F. and L.C.M.

Optional—Finding of H.C.F. by the method of differences, limited to polynomials of degree 3 or less.

Operations with fractions. Excessive manipulative work is to be avoided. Simplification of complex fractions (of reasonable difficulty), by multiplication of numerator and denominator by L.C.M. of individual denominators, should be emphasized.

Equations involving fractions, their simplification and solution. Undue emphasis on specialized forms of fractional equations should be avoided.

Section 4. **Powers, Roots and Surds**

Formation, by inspection, of the terms in the expansion of such expressions as $(x+2y-3z)^2$, $(x^2+2x+3)^2$.

Finding, by inspection, square roots of trinomials which are complete squares; also of such other squares as $a^2+9b^2+4c^2-6ab+4ac-12bc$.

Formal method of finding square roots of algebraic expressions which are perfect squares. Calculation of square roots of numbers.

Quadratic surds. Distinction between rational numbers and surds.

Use of Pythagorean theorem to construct lines of length 22″, 23″, 25″, etc.

Operations with quadratic surds, including rationalizing of denominators.

The square-root table.

Solution of equations with surd coefficients.

Problems, involving surds, on the perimeters and areas of triangles and polygons.

Use of the formula for the area of a triangle in terms of the sides.

Section 5. **Ratio, Proportion, and Elementary Variation**

It is intended that the major portion of the work on these topics should be taken in conjunction with the work on similar triangles in the course in Geometry. Where the Geometry is studied in Grade XII and Algebra in Grade XI, the work on this section may be restricted in Grade XI, to

Definition of ratio, proportion, mean proportional.

The following results, with related exercises:

aK, bK will represent any two numbers in the ratio a : b;

$\frac{a}{b}=\frac{ma}{mb}$; if $\frac{a}{b}=\frac{c}{d}$ then $ad=bc$, $\frac{b}{a}=\frac{d}{c}$, $\frac{a}{c}=\frac{b}{d}$; also $a=bK$ and $c=dK$.

Meaning of direct, inverse and joint variation, with easy problems.

Note: Problems designed merely to give practice in manipulation should be treated as optional. The objective is to give the pupils familiarity with the principles of ratio, proportion and variation so that they may be able to apply them to problems in Algebra, Physics and Chemistry.

Section 6. **Quadratic Equations**

Examples of quadratic equations with numerical and with literal coefficients.

Solution by factoring and by completing the square.

Solution of the general quadratic equation.

Solution, by formula, of quadratic equations possessing real roots.

Graphs of quadratic functions of one unknown as illustrative of the function and as a method of determining the roots of the corresponding equation.

Graded problems leading to quadratic equations.

Section 7. **Quadratic Equations in Two Unknowns**

Solution of a system of two equations of which one is linear and one is quadratic.

Solution of such systems of two quadratic equations as are reducible to systems of the preceding type.

Graphical solution of a pair of equations in two unknowns, one of which is linear and one quadratic, particularly the circle, parabola and rectangular hyperbola.

Problems leading to systems of equations of the types of this section.

Section 8. **Theory of Quadratics**

The discriminant of a quadratic equation; its use in determining the existence or non-existence of real roots; condition for equal roots. The sum and the product of the roots of a quadratic equation; their use as a check on the roots, and in forming an equation with given roots.

Optional: 1. Symmetrical functions of the roots.

2. Problems involving the obtaining of an equation with roots related to those of a given equation.

Section 9. **Indices**

Index laws for positive integral indices.

Definition and use of powers with fractional, zero and negative indices.

Section 10. **Logarithms**

Definition of logarithm.

Use of tables for finding the logarithm of a given number and for finding the number having a given logarithm.

Use of logarithms for calculating products, quotients, powers and roots.

Problems leading to computations in which the use of logarithms is advantageous. Extensive and continuing practice is desirable.

Solution of such equations as $3^x=80$.

Optional: The Slide Rule.

Section 11. **Surds and Surd Equations**

Further examples on quadratic surds.

Surds of higher order than the quadratic. Simple examples.

Simple surd equations resulting in linear or quadratic equations. Testing the roots of the resulting equations as possible roots of the surd equations.

Examples in surd equations should be limited to those involving not more than two unlike surds, with or without a constant. The more complicated examples should be avoided.

Section 12. **Arithmetic and Geometric Series**

Fundamental theorems—nth term and the sum of n terms—and simple problems based on these formulae.

Arithmetic and geometric means.

Optional: The infinite geometric series.

GEOMETRY

The work of Grade X reviewed and extended to include the following propositions, together with deductions based thereon, and a discussion of the topic of ratio, proportion and variation with geometric applications:

Lines parallel to the same straight line are parallel to each other.

The straight line which joins the middle points of two sides of a triangle is parallel to the third side and equal to one-half of it.

The straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side.

To divide a given straight line into any number of equal parts.

The sum of the angles of a polygon of n sides is (2n—4) right angles.

If two sides of one triangle are respectively equal to two sides of another triangle and the third sides unequal, the triangle which has the greater third side has the greater contained angle.

If two equal triangles are on the same side of a common base, the straight line which joins their vertices is parallel to the base.

The complements of the parallelograms about the diagonal of a parallelogram are equal.

To construct a parallelogram equal to a given triangle and having an angle equal to a given angle.

To construct a triangle equal to a given rectilineal figure.

To construct a triangle equal to a given triangle, and having one of its sides equal to a given straight line.

The square on the sum of two straight lines is equal to the sum of the squares on the two lines increased by twice the rectangle contained by them. (Algebraic proof only.)

The square on the difference of two straight lines is equal to the sum of the squares on the lines, diminished by twice the rectangle contained by them. (Algebraic proof only.)

The difference of the squares on two straight lines is equal to the rectangle contained by the sum and the difference of the two lines. (Algebraic proof only.)

If the square on one side of a triangle is equal to the sum of the squares on the other two sides, the angle contained by these two sides is a right angle.

The locus of points on one side of a given straight line and at a given distance from that line is a straight line parallel to the given line, through any point at the given distance from the given line.

The locus of a point which is equidistant from two fixed points is the right bisector of the straight line joining the two points.

The locus of points which are equidistant from two given intersecting straight lines is the pair of lines which bisect the angles between the given lines.

The right bisector of any chord of a circle passes through the centre.

To find the centre of a given circle.

To circumscribe a circle about a triangle.

Chords of a circle that are equally distant from the centre are equal.

Equal chords in a circle are equally distant from the centre.

Of two chords in a circle, the one which is nearer the centre is greater than the one which is more remote.

Of two unequal chords in a circle, the greater is nearer the centre than the less.

An angle at the centre of a circle is double an angle at the circumference standing on the same arc.

Angles in the same segment of a circle are equal.

If the straight line joining two points subtends equal angles at two other points, on the same side of it, the four points are concyclic.

The angle in a semicircle is a right angle.

The circle described on the hypotenuse of a right-angled triangle as diameter passes through the vertex of the right angle.

The opposite angles of a quadrilateral inscribed in a circle are supplementary.

If a pair of opposite angles of a quadrilateral are supplementary, its vertices are concyclic.

The straight line drawn perpendicular to a radius of a circle, at the circumference, is a tangent.

To draw a tangent to a circle from a given point outside the circle.

If two tangents are drawn to a circle from an external point, (1) the two tangents are equal, (2) they subtend equal angles at the centre, (3) they make equal angles with the straight line joining the point to the centre.

To inscribe a circle in a triangle.

To draw the escribed circles of a triangle.

To inscribe a circle in a given regular polygon.

About a given circle to circumscribe a triangle, equiangular to a given triangle.

If from the point of contact of a tangent to a circle a chord be drawn, each of the angles between the chord and the tangent is equal to the angle in the segment on the other side of the chord.

On a given straight line, to describe a segment of a circle, containing an angle equal to a given angle.

In a given circle, to inscribe a triangle equiangular to a given triangle.

If two circles touch each other, their line of centres passes through the point of contact.

Arcs of a circle which subtend equal angles at the centre are equal.

The areas of triangles of equal altitude are proportional to their bases.

A straight line drawn parallel to one side of a triangle divides the other two sides proportionally.

If two sides of a triangle are divided in the same ratio, the straight line joining the points of section is parallel to the third side.

To find the fourth proportional to three given straight lines.

To divide a given straight line (1) internally, (2) externally, in a given ratio.

If two triangles are equiangular to each other, their corresponding sides are proportional and hence the triangles are similar.

If two triangles have the sides of one proportional to the sides of the other, they are equiangular to each other, and hence are similar.

If two triangles have an angle of one equal to an angle of the other, and the sides about these angles proportional, the triangles are similar.

The areas of similar triangles are proportional to the squares on corresponding sides.

The perpendicular to the hypotenuse of a right-angled triangle from the opposite vertex is a mean proportional between the segments of the hypotenuse, and each of the sides about the right angle is a mean proportional between the hypotenuse and the adjacent segment of the hypotenuse.

To find a straight line which is a mean proportional between two given straight lines.

To construct a square equal in area to a given rectilineal figure.

If two chords of a circle intersect, the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other.

If two straight lines intersect so that the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other, the ends of the lines are concyclic.

If from a point outside a circle a tangent be drawn to the circle, and also a secant, the square on the tangent is equal to the rectangle contained by the secant and the part of it outside the circle.

If from a point outside a circle, two straight lines be drawn, one of which meets the circle, and the other is a secant, and if the square on the line which meets the circle is equal to the rectangle contained by the secant and the part of it outside the circle, the line which meets the circle is a tangent.

The bisector of any angle of a triangle divides the opposite side into segments which have the same ratio as the sides about that angle.

If one side of a triangle is divided into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the angle at that vertex.

The bisector of the exterior angle at any vertex of a triangle divides the opposite side externally into segments which have the same ratio as the other two sides.

If one side of a triangle is divided externally into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the exterior angle at that vertex.

In equal circles, angles at the centre are proportional to the arcs on which they stand.

On a given straight line to construct a polygon similar to a given polygon.

Similar polygons can be divided into similar triangles.

The areas of similar polygons are proportional to the squares on corresponding sides.

In a right-angled triangle, any rectilineal figure described on the hypotenuse is equal to the sum of the similar and similarly described figures on the other two sides.

In an obtuse-angled triangle, the square on the side opposite the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle, increased by twice the rectangle contained by either of these sides and the projection of the other side on it.

In any triangle, the square on the side opposite an acute angle is equal to the sum of the squares on the sides containing that angle, diminished by twice the rectangle contained by one of these sides and the projection of the other side on it.

In any triangle the sum of the squares on two sides is equal to twice the square on half the third side increased by twice the square on the median to the third side.